

Growth Rate and Growth Factor Transcript

Instructional Coach: Hi, thanks for coming in today. We are planning for next week when we will focus on exponential functions. It's helpful that we have Algebra I and Algebra II represented in our group so that we can use vertically aligned academic language. Will someone read aloud student expectations A(9)(B) and A(9)(C)?

Teacher 1: Sure, A(9)(B) states the student is expected to interpret the meaning of the values of a and b in exponential functions of the form $f(x) = (ab)^x$. A(9)(C) states the student is expected to write exponential functions in the form of $f(x) = (ab)^x$, where b is a rational number, to describe problems arising from mathematical and real-world situations, including growth and decay.

Instructional Coach: We have talked about functions as well as their domain and range. We have also looked at the graphs of functions. Now, we will explore the meaning of a and b when writing exponential functions. So when you think about exponential functions, what real-world situations come to mind?

Teacher 2: There are always the questions about bacterial growth, such as bacteria doubling every hour.

Teacher 1: Or the questions about decay, like the half-life problems. I have also seen questions about the growth of a population or the depreciation of a car's value.

Teacher 2: Last year, I taught exponential functions when the value increased by a percentage but had to reteach it. My students struggled with this concept.

Instructional Coach: What were their misconceptions or challenges?

Teacher 2: For the doubling problems, my students understood to use two as the b value. However, when the values were growing by a percentage, like five percent each year, many students used five for the b value. Some students would change the percent to a decimal and use 0.05. Either way they weren't getting the correct answer.

Instructional Coach: It sounds like they were having trouble representing the growth rate as a decimal value.

Teacher 1: My students understood how to represent a percentage as a decimal, but they didn't understand why they were adding the one to determine the value of b .

Teacher 2: Do you think students missed the connection between the growth rate and the growth factor? It sounds like they struggled to remember to add the one because they still had 100 percent of their original amount plus another five percent. So I'm wondering...

Teacher 1: (*Interrupts.*) Wait, wait, growth factor? I didn't use that term in my class.

Teacher 2: The factor is the value of b . The rate is the percent increase or decrease.

Teacher 1: I didn't use those terms. I just said that if the value is increasing by five percent each year, then I use one plus five percent for b . If the value is decreasing by five percent then I use one minus five for b . I didn't put a name to it. Is that what you are talking about when you say growth factor?

Growth Rate and Growth Factor Transcript

Teacher 2: Yes. When the value of b is greater than one, it is a growth factor; and when the value is less than one, it's a decay factor.

Teacher 1: My students always ask, "Why I am adding the one?" How do I answer that question?

Instructional Coach: The one accounts for the current value, but let's look at a specific example. When a population is doubling, the growth factor would be two. Students tend to make the connection between doubling and a growth factor of two. Using this understanding can help us build the meaning of adding one to determine a growth factor. We will use the function $f(x) = 100(2)^x$, where b , the growth factor, is two. Let's look at this table to track what happens (*displays table*). The initial value of 100 can be seen when $x=0$. Students can confirm that $f(x)$ is 100 when x is zero. When $x=1$, we can record the input into the function (*displays third row on table*). What do they notice when they compare the output for $x=1$ to the output for $x=0$?

Teacher 2: Well, I hope my Algebra II students notice that the output for $x=1$ is twice that for $x=0$, but why did you write $100 \cdot 1 \cdot 2$?

Instructional Coach: Well 100 is our initial value. The one comes from this initial iteration. The two, of course, doubles it. (*Displays fourth row on table*.) What do they notice when they compare the output for $x=2$ to the output for $x=1$?

Teacher 2: I hope they notice that the output for $x=2$ is twice that for $x=1$.

Instructional Coach: We can help students see that the previous amount is doubled by writing an equivalent expression. (*Refers to table*.) The portion in the parentheses represents the previous value. The pattern continues as you complete the table.

Teacher 1: So, the initial value is 100, and the growth factor is two. What is the growth rate?

Instructional Coach: To answer that question, let's look at the table of values for this function. Up to this point, we have looked at the process used to determine the output. Let's calculate the outputs.

Teacher 2: Doubling?

Instructional Coach: Think of it through the lens of rate of change. How would you describe the change in $f(x)$ as x increases by one? (*Gestures to table*.) As x increases by one, from zero to one, $f(x)$ increases by 100, from 100 to 200. As x increases from one to two, $f(x)$ increases by 200, from 200 to 400. As x increases from two to three, $f(x)$ increases by 400, from 400 to 800. If we fold the paper to put values for x in their corresponding values of $f(x)$ immediately next to each other, we can see the doubling you mentioned while focusing on the x and $f(x)$ paired values on each row of the table.

Teacher 1: Well, if we look at how $f(x)$ is increasing, when x changes from zero to one, $f(x)$ increases by 100. If we look at how $f(x)$ is increasing, when x changes from one to two, $f(x)$ increases by 200, and then, if we look at how $f(x)$ is increasing when x changes from two to three, $f(x)$ increases by 400.

Growth Rate and Growth Factor Transcript

Teacher 2: That shows that the function does not have a constant rate of change. What's another way we could think about how the function values are changing?

Instructional Coach: Great question. How could you use the pattern to determine the next value in the table?

Teacher 1: Looking at the pattern, you would have to add 800 to the previous value to get the next value. So, it would be $x=4$ and $f(x)=1600$.

Teacher 2: Right, and the next value to be added would be 1600. Each time, the value added is doubled.

Teacher 1: Another way you could look at it is the current value is added to itself, or doubled, to get the next value.

Instructional Coach: Let's explore that a little further. The pattern you noticed is that the current value is added to itself to get the next value. One of the ones represents the current value. We are adding 100 percent of the current value to determine the next value.

Teacher 1: That makes sense. I see why the growth factor in this case is two. It is 100 percent of the initial value plus 100 percent more which equals two.

Teacher 2: The same should be true looking at the situation with five percent growth. Can we make a table and see if we can see the same pattern?

Instructional Coach: I've made this table (*displays new table*). Let's look at a five percent growth with an initial value of 100. This would be modeled by the function $f(x)=100(1.05)^x$.

The initial value of 100 can be seen when $x=0$ (*displays second row on table*). (*Displays third row on table*.) Students can confirm that $f(x)$ is 100 when $x=0$. When $x=1$, we can record the input into the function. What do they notice when they compare the output for $x=1$ to the output for $x=0$?

Teacher 2: I hope my Algebra II students notice that the output for $x=1$ is 1.05 that for $x=0$, but why did you write $100 \cdot 1.05$?

Instructional Coach: The 100 is our initial value. The one comes from this initial iteration (*displays fourth row on table*). The 1.05, of course is the amount of the increase. What do they notice when they compare the output for $x=2$ to the output for $x=1$?

Teacher 2: I hope they notice that the output for $x=2$ is five percent more than that for $x=1$.

Instructional Coach: We can help students see that the previous amount is increased by 5 percent by writing an equivalent expression. (*Refers to table*.) The portion in the parentheses represents the previous value. The pattern continues as you complete the table (*displays fifth row on table*).

Teacher 1: So, the initial value is 100, and the growth factor is 1.05. What is the growth rate? To examine the change in $f(x)$ when x increases by one, let's note how much $f(x)$ increases as we increase x by one and see if we notice the same type of pattern.

Teacher 2: I want to say that any amount is five percent more than the previous amount, so five percent is the growth rate. To find five percent of a value, I multiply the value by

Growth Rate and Growth Factor Transcript

0.05. For example, $f(x)$ increases by five when x increases from zero to one. Five percent of the current value, 100, is five, and $f(x)$ increases by 5.25 when x increases from one to two. Five percent of 105 is 5.25. The same is true when x increases from two to three. $F(x)$ increases by 5.5125, which is five percent of the value 110.25.

Teacher 2: So we could use the same idea of one plus the growth rate to determine the growth factor. The current value, plus the percent of increase or growth rate. Taken together, the current value plus the growth rate gives us the growth factor.

Instructional Coach: How do you think determining the growth rate would change if you were writing a function to represent exponential decay?

Teacher 2: I think that instead of adding the growth rate to one, I would subtract it from the one, since that would represent the amount the value decreases with each increment of x .

Teacher 1: That makes sense. So, an exponential function that represents exponential decay of five percent for each increment would have a decay factor of $1-0.05=0.95$.

Teacher 2: We could say that 95 percent of the amount remains after each increment of x .

Instructional Coach: Looking at it this way could help students see why the factor is determined by one plus or minus the rate. When we have exponential growth, the growth factor is one plus the growth rate. When we have exponential decay, the decay factor is one minus the rate of decay.