Section 6: Probability

The following maps the videos in this section to the Texas Essential Knowledge and Skills for Mathematics TAC §111.47(c).

6.01 Probability and the Law of Large Numbers

- Statistics (1)(F)
- Statistics (3)(C)
- Statistics (5)(A)
- Statistics (5)(B)

6.02 Probability Terminology

- Statistics (5)(A)

6.03 Venn Diagrams

- Statistics (1)(D)
- Statistics (5)(A)

6.04 Independent and Mutually Exclusive Events

- Statistics (1)(F)
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6.05 Contingency Tables

- Statistics (1)(D)
- Statistics (1)(F)
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- Statistics (5)(A)

6.06 Tree Diagrams

- Statistics (1)(D)
- Statistics (5)(A)

6.07 Discrete Random Variables

- Statistics (1)(F)
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6.08 The Binomial Distribution

- Statistics (1)(F)
- Statistics (5)(A)

6.09 Binomial Approximation

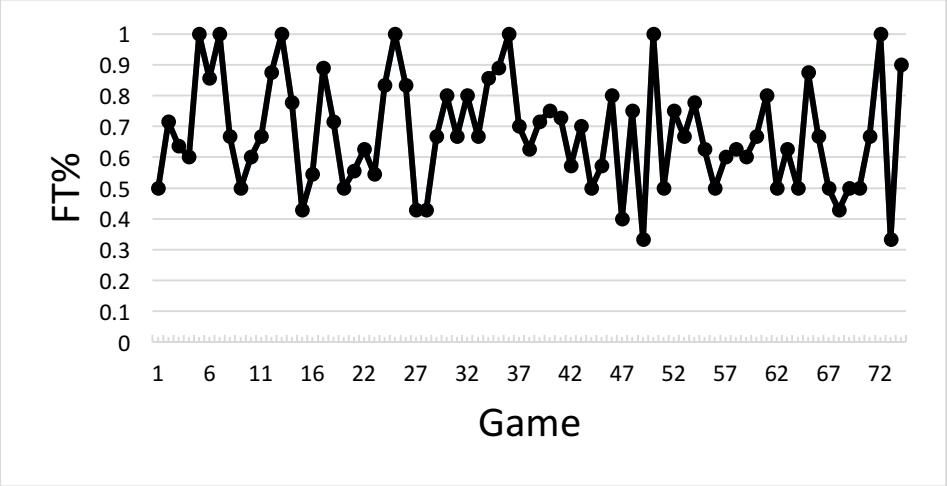
- Statistics (1)(F)
- Statistics (5)(A)

Note: Unless stated otherwise, any sample data is fictitious and used solely for the purpose of instruction.

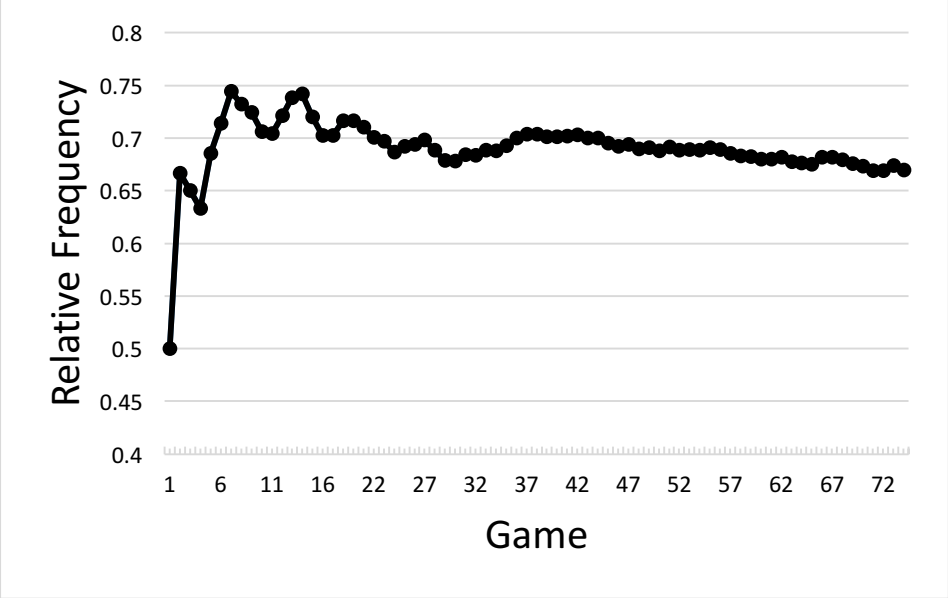
6.01
Probability and the Law of Large Numbers

Will LeBron James make his next free throw? We do not know the ***outcome*** of the next ***trial***, but in the long run a predictable pattern emerges.

Consider the graph of LeBron's free throw percentage (FT%) during the 2016–2017 season ("LeBron James," n.d.).



Rather than looking at the percentage made per game, we can look at the relative frequency to determine the probability that LeBron will make a given free throw.



What is probability?

- The next outcome is unknown.
- A predictable pattern does emerge in the _____ run.
- The probability of an event must be between _____ and _____.

Based on the graphs and data, the relative frequency is about 67%; the probability that LeBron will make his next free throw is about _____.

This follows the ***Law of Large Numbers (LLN)***.

- As the number of trials increases, the relative frequency obtained becomes closer and closer to the theoretical probability.
- $P(A) = \frac{\text{\# times outcome A occurs}}{\text{\# trials}}$

1. How can you find the probability that a coin will land on heads when flipping it?

6.02
Probability Terminology

Consider the case of flipping two fair coins. Let A be the event of flipping at least one heads, and B be the event of flipping at least one tails.

Terminology

- ***Sample space*** – The collection of all possible outcomes. A sample space can be visualized by listing the outcomes using a Venn diagram, contingency table, or tree diagram.

- ***Outcome*** – The result of a trial

- ***Event*** – A subset of the sample space

- ***Equally likely*** – Describes outcomes that have the same probability of occurring

- ***Complement***
  - The complement of event A is all the outcomes that are _____ A.
  - The complement of A is denoted A' or A^c.

- ***Union***
  - The union of A and B is the collection of all outcomes in A or B or both.
  - The union of A and B is an "or" event, also denoted as $A \cup B$.

- ***The Addition Rule*** – The probability for the union of A and B, written as $P(A \cup B) =$

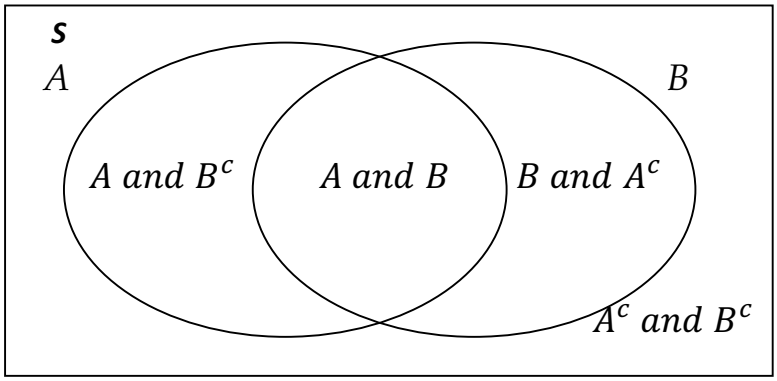
- ***Intersection***
  - The intersection of A and B is the shared collection of outcomes that are in both A and B.
  - The intersection of A and B is an "and" event, also denoted as $A \cap B$.

- ***Conditional probability***
    - The conditional probability of A given B is the probability that event A will occur given that event B has already occurred.
    - Conditional probability restricts the sample space to the given outcomes.
    - $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\#\ outcomes\ in\ A\ and\ B}{\#\ outcomes\ in\ B}$

- ***The Multiplication Rule*** – The probability that A and B both occur, written as $P(\mathrm{A} \cap \mathrm{B}) =$

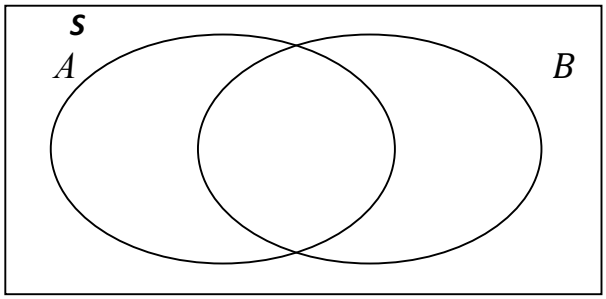
6.03
Venn Diagrams

Venn diagram – A display used to show probabilistic outcomes

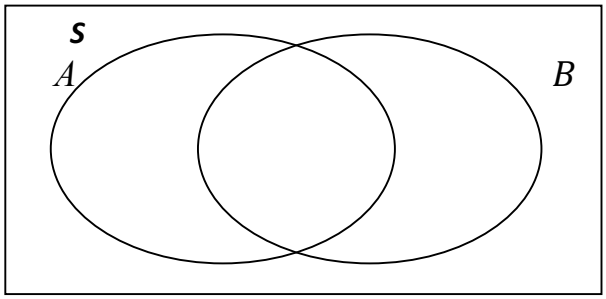


Shade the following probabilities:

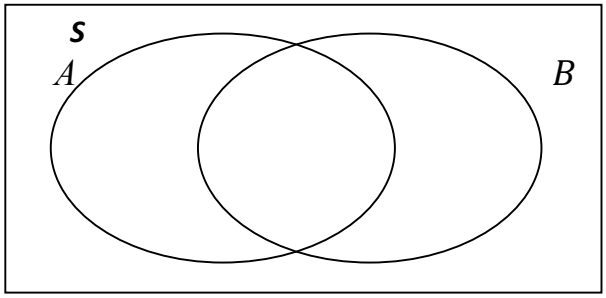
1. $P(A)$



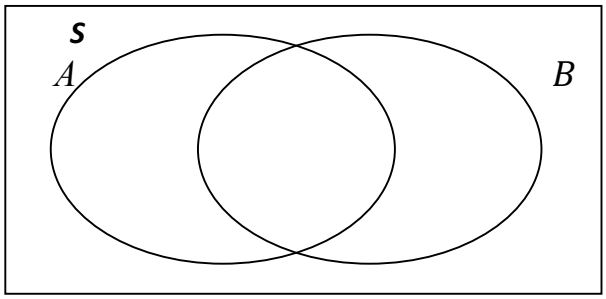
2. $P(A^c)$



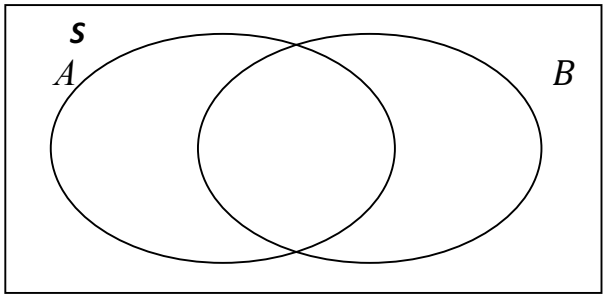
3. $P(A \text{ and } B) = P(A \cap B)$



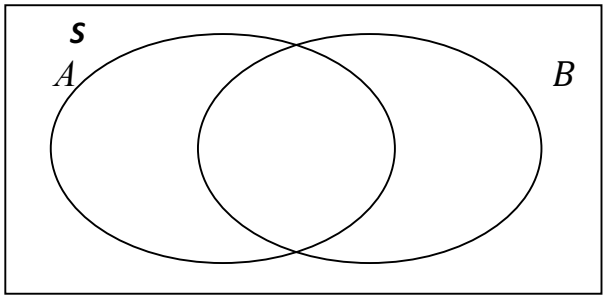
4. $P(A \text{ or } B) = P(A \cup B)$



5. $P(A^c \text{ and } B) = P(A^c \cap B)$



6. $P(A^c \text{ and } B^c) = P(A^c \cap B^c)$



6.04
Independent and Mutually Exclusive Events

Mutually exclusive (disjoint)

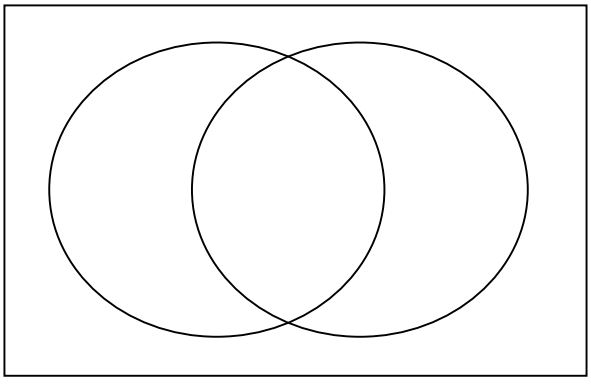
- Two events are mutually exclusive if they cannot occur at the same time.
- $P(A \cap B) =$
- $P(A \cup B) =$

Independence

- Two events are independent if the occurrence of one does not affect the probability of occurrence of the other.
- $P(A|B) =$
- $P(A \cap B) =$

Suppose that at Texas A&M University, 50% of students attend at least one home football game (F), 15% of students attend at least one home basketball game (B), and 10% of students attend at least one of each.

1. Appropriately label the Venn diagram and fill in the table shown below.



Outcome	Probability
$F \cap B$	
$F \cap B^C$	
$F^C \cap B$	
$F^C \cap B^C$	

2. What is the probability a randomly selected college student does not attend any home football games?

3. What is the probability a randomly selected college student attends at least one home football game or attends at least one home basketball game?

4. What is the probability a randomly selected college student attends at least one home football game, or attends at least one home basketball game, but not both?

5. If a randomly selected college student attends at least one home football game, what is the probability that (s)he attends at least one home basketball game?

6. Are F and B mutually exclusive?

7. Are F and B independent?

6.05
Contingency Tables

Below is a contingency table showing the observed count for the variables "Survived" and "Class of Ticket" on the Titanic ("Sinking," 2017). Let F and S be defined as follows:

F = passenger had a first-class ticket S = passenger survived

	Survived		
Class	No	Yes	Total
1st	122	202	324
2nd	166	118	284
3rd	531	178	709
Total	819	498	1317

1. What percentage of passengers had first-class tickets?

2. What percentage of passengers did not have first-class tickets?

3. What percentage of passengers survived?

4. What percentage of passengers had first-class tickets and survived?

5. What percentage of passengers had first-class tickets or survived?

6. Given that a passenger had a first-class ticket, what is his or her chance of survival?

7. What percentage of passengers who survived had first-class tickets?

8. Are F and S mutually exclusive?

9. Are F and S independent?

6.06
Tree Diagrams

Suppose Jasmine makes 85% of her first free throw attempts. If she makes her first free throw, there is a 90% chance that she will make her second. If she misses her first free throw, there is a 60% chance that she will make her second. Use a tree diagram to answer the following questions.

1. What is the probability that Jasmine will make both free throws?

2. What is the probability that Jasmine won't make either free throw?

3. What is the probability that Jasmine will make at least one free throw?

6.07
Discrete Random Variables

A friend wants to play a game with you. You have to pay $5 to play the game. If you roll doubles (1 and 1, 2 and 2, etc.), you win $10. If you roll a sum of 11 you win $20. If you roll anything else, you lose.

Your winnings are an example of a ***random variable*** because we do not know the outcome.

To determine your ***expected value***, we use the ***probability distribution*** for the random variable.

		Die1					
		1	*2*	*3*	*4*	*5*	*6*
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
Die2	*3*	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

1. Determine the probability distribution of your potential winnings.

Outcome	Winnings	Probability
Doubles		
11		
Anything else		

Mean and Variance of a Discrete Random Variable

- Expected value (mean): $\mu_X = E(X) = \sum xP(x) = x_1P_1 + \cdots + x_nP_n$

- Variance: ${\sigma^2}_X = Var(X) = \sum(x - \mu)^2P(x) = x_1P_1 + \cdots + x_nP_n$

2. What are your expected winnings? Should you play the game?

3. What is the variance of your expected winnings?

6.08
The Binomial Distribution

Suppose the probability that LeBron James makes any free throw is 67%, and his free throws are independent. This situation is called a ***Bernoulli trial***.

Characteristics of Bernoulli Trials

- There are two possible outcomes: _____ and _____.
- The probability of success, p, is _____.
- The trials are _____.

Suppose LeBron is going to shoot 10 free throws. The probability that he will make a certain number of shots—for example, seven shots, $P(X) = 7$—is an example of a ***binomial probability***.

Characteristics of the Binomial Model

- $n =$ number of Bernoulli trials
- $p =$ probability of success
- $E(X) = np$
- $\sigma(X) = \sqrt{npq}$, where $q = 1 - p$
- $P(X = x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$, where $\binom{n}{x} = C_x^n = \frac{n!}{x!(n-x)!}$

1. How many shots is LeBron expected to make?

2. What is the standard deviation of the number of shots LeBron will make?

3. What is the probability he will make exactly seven shots?

4. What is the probability he will make at least one shot?

6.09
Binomial Approximation

Suppose LeBron James shoots 100 independent free throws, each with $p = 0.67$.

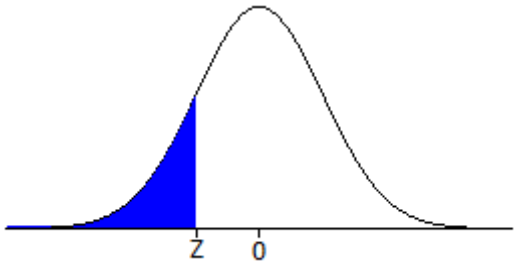
1. What is the distribution of X, the number of free throws LeBron makes?

2. What is the probability that he will make more than 70 shots?

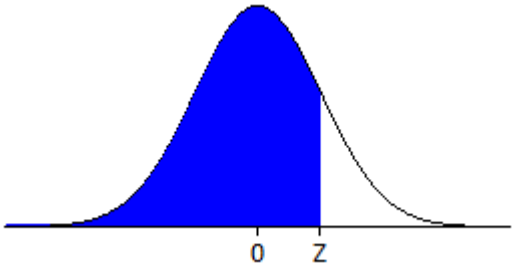
If n is large, if _____ and _____, then the binomial distribution can be approximated with the _____, with a mean equal to _____ and a standard deviation of _____.

3. Using the normal distribution to approximate a binomial, find the probability that LeBron will make more than 70 of his free throws.

Z table



	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641



	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

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