Section 7: Sampling Distributions

The following maps the videos in this section to the Texas Essential Knowledge and Skills for Mathematics TAC §111.47(c).

7.01 Variability in Sample Proportions – Part 1

- Statistics (3)(A)
- Statistics (3)(B)
- Statistics (3)(C)
- Statistics (3)(D)

7.02 Variability in Sample Proportions – Part 1

- Statistics (3)(A)
- Statistics (3)(B)
- Statistics (3)(C)
- Statistics (3)(D)
- Statistics (5)(D)

7.03 Variability in Sample Means

- Statistics (3)(A)
- Statistics (3)(B)
- Statistics (3)(C)
- Statistics (3)(D)
- Statistics (5)(D)

7.04 Using the Central Limit Theorem

- Statistics (5)(D)

Note: Unless stated otherwise, any sample data is fictitious and used solely for the purpose of instruction.

7.01
Variability in Sample Proportions – Part 1

Assume that of adults who follow at least one sport, the proportion who identify professional football as their favorite is 33%.

1. If 50 adults are randomly sampled from the population, what proportion should you expect to choose professional football as their favorite?

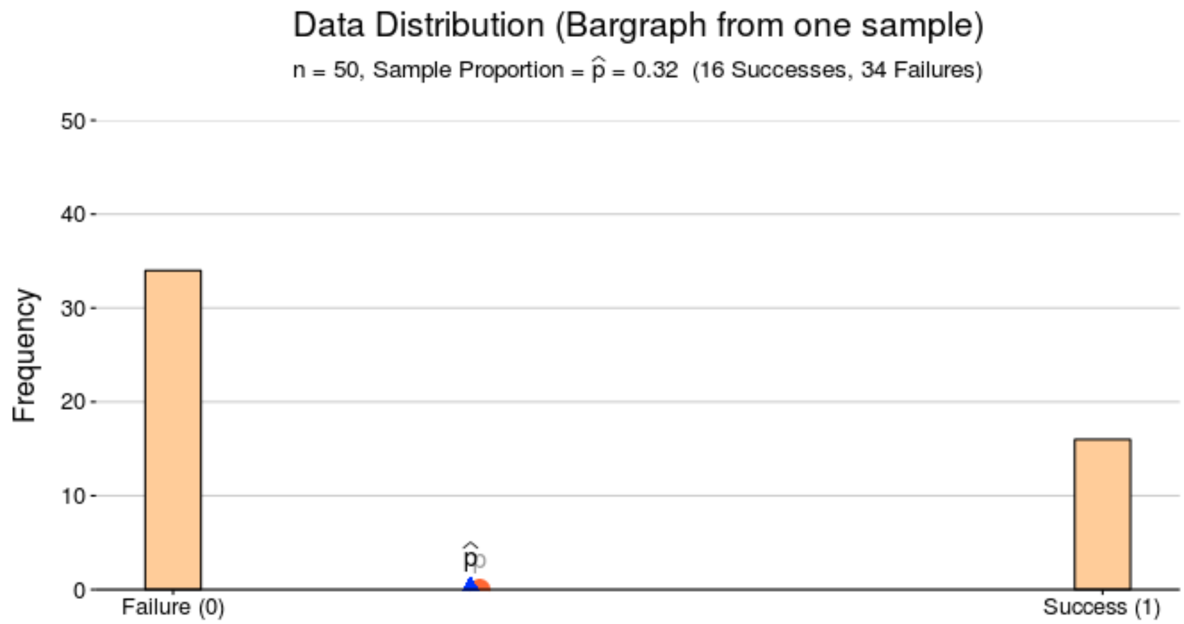
2. What if a different group of 50 adults is sampled?

The variability observed from one sample to another is called ***sampling error***.

Suppose that in a sample of 50 randomly selected adults, 16 identified professional football as their favorite sport.

1. What do you notice about the population distribution?

2. What do you notice about the data distribution?

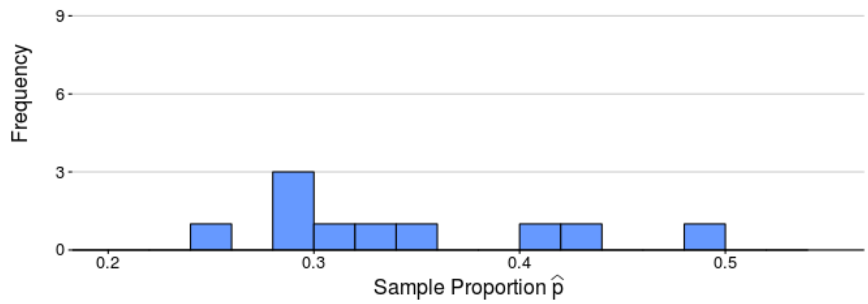


7.02
Variability in Sample Proportions – Part 2

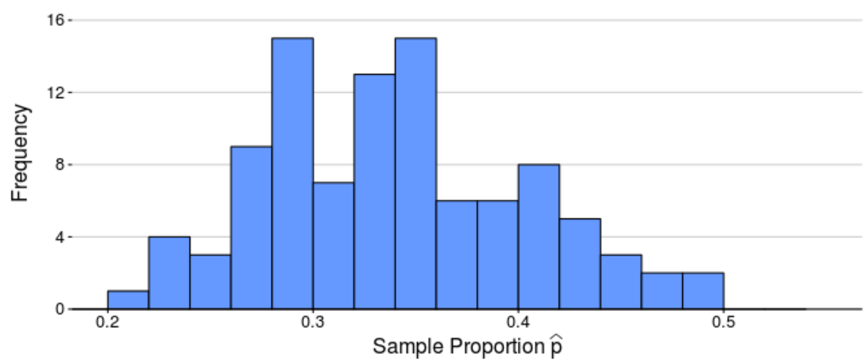
Assume that of adults who follow at least one sport, the proportion who identify professional football as their favorite is 33%. In a sample of 50 randomly selected adults, 16 identify professional football as their favorite sport.

Suppose random samples of size 50 were taken 10 times, 100 times, and 1,000 times.

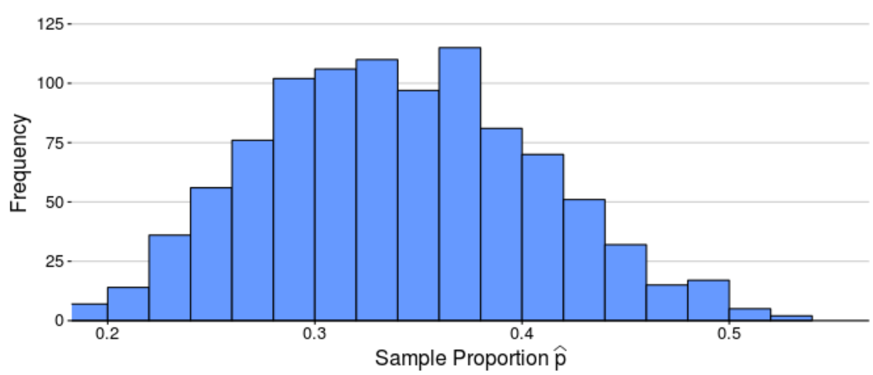
10 simulations of size $n = 50$



100 simulations of size $n = 50$



1,000 simulations of size $n = 50$



1. In the histograms above, what do you notice about the shape of the distribution of $\hat{p}$?

The ***sampling distribution*** of a statistic refers to all possible values of the statistic in _______ possible samples of the same size.

- The sampling distribution of $\hat{p}$ is approximately normal when $np \geq 15$ _______ $n(1-p) \geq 15$.
- $\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$

2. Based on the histograms above, what is the sampling distribution of $\hat{p}$?

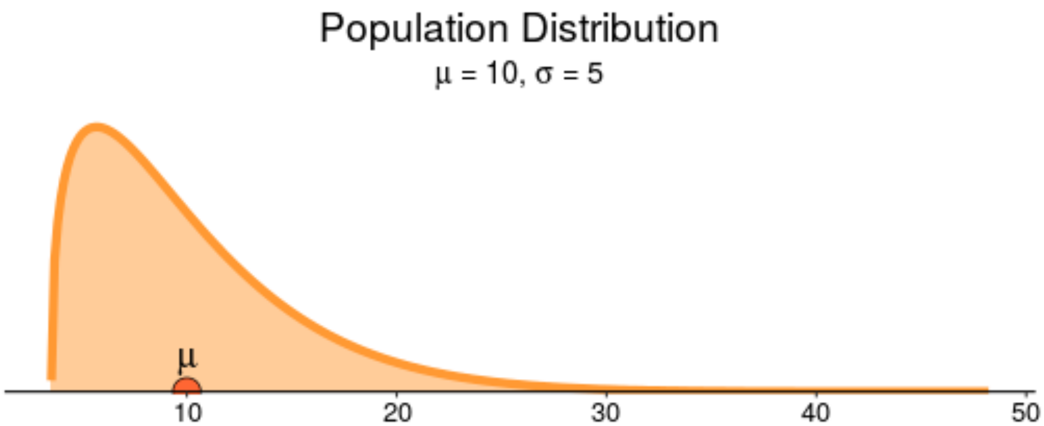
STUDY EDGE TIP

The population distribution and data distribution of a proportion are never normal.

7.03
Variability in Sample Means

How much cash do you usually carry?

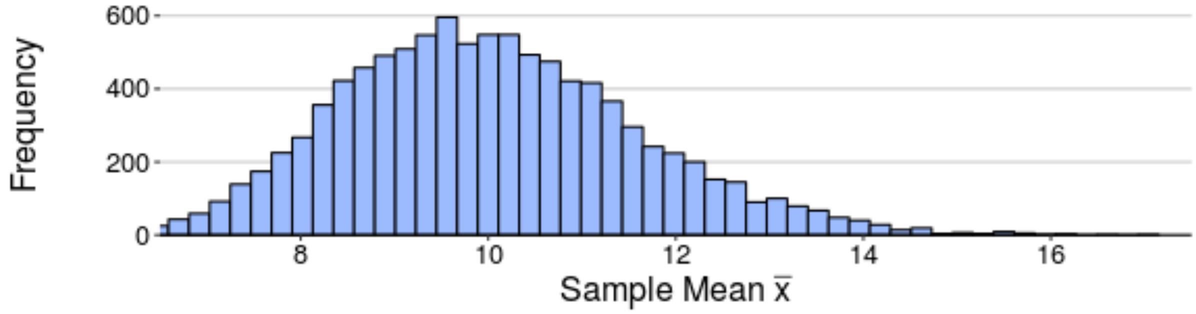
Assume that the distribution of the amount of cash students have on hand is skewed to the right with a mean of $10 and a standard deviation of $5.



1. If a random sample of 10 students is selected, what should you expect the average amount of cash each is carrying to be? What if you sampled another 10 students?

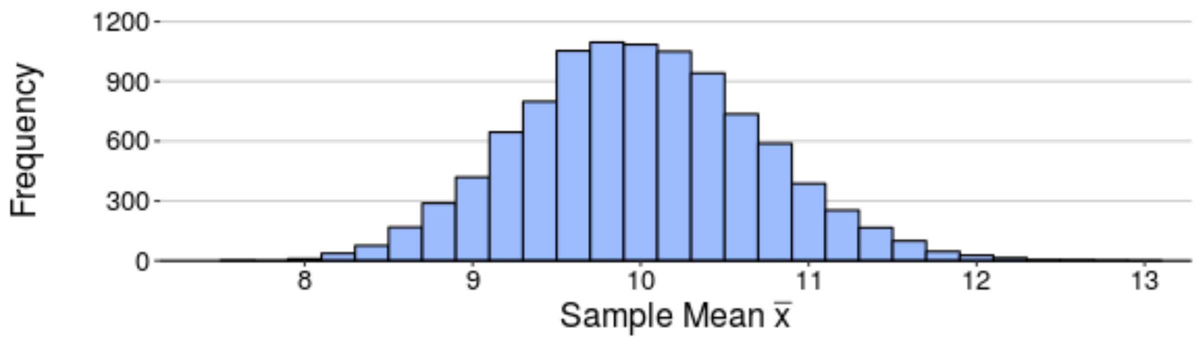
2. If we were to repeat this sampling 10,000 times, what would the histogram of these sample means look like?

10,000 simulations of size $n = 10$



Now, suppose we select repeated random samples of size $n = 50$.

10,000 simulations of size $n = 50$



3. What do you notice about the sample means?

The ***central limit theorem (CLT)*** states that the distribution of the sample mean will be approximately normal, regardless of the shape of the population distribution, if the sample size is sufficiently large.

- The sampling distribution of any mean will approach a normal distribution as the sample size increases.
- If the population is normally distributed or the sample size is sufficiently large, $\bar{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.

7.04
Using the Central Limit Theorem

Assume that the distribution of the amount of cash students carry is skewed to the right, with a mean of $10 and a standard deviation of $5.

1. What does the CLT predict about the mean amount of money in random samples of 30 students?

2. What is the distribution of the sample mean?

3. Use the 68–95–99.7 rule to make conclusions about the average amount of cash seen in random samples of 30 students.